March 24, 2026

The kick-off meeting of our Action took place from March 16 to March 19 at Bocconi University in Milan. Hosted by the Bocconi Institute for Data Science and Analytics (BIDSA), the event gathered around fifty Action members for four intensive and stimulating days, providing a valuable opportunity to get to know each other and exchange ideas.
Each day was structured around one of the working groups (WGs), beginning with a roadmap of themes and objectives presented by the WG leaders.
From Monday to Thursday, the program featured each day four to five targeted research talks – with a strong emphasis on junior scientists – alongside two engaging plenary lectures delivered by renowned mathematicians. These lectures covered

- linear and nonlinear waves on metric graphs (WG1);
- spectral geometry of manifolds and polytopes (WG2);
- parabolic equations via nonlinear semigroup method and calculus of variations (WG3);
- tessellation and stochastic modeling of microstructures (WG4).

Complementing the lectures, a poster session and an open problem session offered the chance to discuss the latest findings of our Action members. During the open problem session, participants presented stimulating challenges that we look forward to addressing in the coming years.
Beyond the technical program, participants also had the chance to enjoy Milan's crisp, sunny weather, discussing new ideas and possible future collaborations in a vibrant environment, and we shared a memorable social dinner by the Navigli canals.
We are confident that in four years, we will look back on this meeting in Milan as the starting point of outstanding research and fruitful collaborations.

The meeting was organized by Hugo Lavenant together with Katie Gittins and Delio Mugnolo.

Multiscale Stochastics, Patterns, and Analysis of Combinatorial Environments

Bocconi University (Room AS01)
March 16 – 19, 2026

Monday 16 Analysis of discrete and metric complex systems, and beyond	**Tuesday 17** Spectral geometry of continuous spaces	**Wednesday 18** Probabilistic methods and mathematical foundations of materials science	**Thursday 19** Dynamics on structured spaces
10:00 – 10:30 Registration & Welcome	9:00 – 10:00 Julie Rowlett	9:00 – 10:00 Martina Vittorietti	9:00 – 10:00 José M. Mazón
10:30 – 11:30 Pavel Kurasov & Marjeta Kramar Fijavž	10:00 – 10:30 Davide Buoso	10:00 – 10:30 Daniela Flimmel	10:00 – 10:30 Ozlem Defterli
	10:30 – 11:00 Coffee break	10:30 – 11:00 Coffee break	10:30 – 11:00 Coffee break
11:30 – 12:00 Maria Rosaria Lancia	11:00 – 11:30 Hugo Tavares	11:00 – 11:30 Eviatar B. Procaccia	11:00 – 11:30 Liviu Ignat
12:00 – 12:30 Claudio Cacciapuoti	11:30 – 12:00 Jaume de Dios Pont	11:30 – 12:00 Luca Makowiec	11:30 – 12:00 Antonio Esposito
12:30 – 14:00 Lunch and discussions	12:00 – 13:30 Lunch and discussions	12:00 – 13:30 Lunch and discussions	12:00 – 13:30 Lunch and discussions
14:00 – 15:00 Colette De Coster	13:30 – 14:30 Alexandre Girouard	13:30 – 14:30 Viktor Beneš	13:30 – 14:30 Giuseppe Savaré
15:00 – 15:30 Anna Rozanova-Pierrat	14:30 – 15:00 Gloria Paoli	14:30 – 15:00 Vesna Gotovac Đogaš	14:30 – 15:00 Sara Farinelli
15:30 – 16:00 Coffee break	15:00 – 15:30 Coffee break	15:00 – 15:30 Coffee break	
16:00 – 16:30 Simon Becker	15:30 – 17:00 MC meeting	15:30 – 16:00 Jussi Behrndt	
16:30 – 17:00 Mats-Erik Pistol		16:00 – 17:30 Open problems session	
17:00 – 18:30 Poster session			
18:30 – 20:00 Aperitivo		20:00 – 22:00 Dinner	

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Book of abstracts for the conference

Multiscale Stochastics, Patterns, and Analysis of Combinatorial Environments

Monday March 16, 10:30 – Marjeta Kramar Fijavž and Pavel Kurasov

Analysis on discrete and metric complex systems

We will provide introduction into analysis on discrete and metric graphs: some fundamental results and recent developments. We are particularly interested in the influence of the geometry of the graph to the spectrum of the corresponding differential operator and long-term behavior of the process.

The main goal of our talk is not only to provide solid basis for analysis on metric and discrete graphs, but to outline the list of possible research directions for the first working group (WG1) and open possibilities for collaboration with other working groups.

Monday March 16, 11:30 – Maria Rosaria Lancia

Parabolic Problems in Fractal Domains: Autonomous vs Non-Autonomous – Results and Open Problems

This talk explores the analysis of evolution equations in domains with fractal boundaries, focusing on the interplay between geometric complexity and temporal evolution.

We begin by briefly discussing linear parabolic problems subject to Venttsel boundary conditions, highlighting how pre-fractal approximations provide a fundamental tool for understanding diffusion in such irregular settings. The second part of the presentation shifts toward non-autonomous settings, including problems with or without Venttsel conditions. We examine various scenarios, highlighting the different behaviors that emerge when spatial irregularity meets time-dependent operators. In this context, we discuss the challenges related to well-posedness and the limitations of standard approximation techniques, providing a concise overview of current results and open analytical questions.

Monday March 16, 12:00 – Claudio Cacciapuoti

The Born-Oppenheimer approximation for a 1D 2+1 particle system with zero-range interactions

We study the self-adjoint Hamiltonian that models the quantum dynamics of a one-dimensional (1D) three-body system consisting of a light particle interacting with two heavy ones through a zero-range force. For an attractive interaction we determine the behavior of the eigenvalues below the essential spectrum in the regime $\varepsilon \ll 1$, where ε is proportional to the square root of the mass ratio. We show that the n-th eigenvalue behaves as $E_n(\varepsilon) = -\alpha^2 + |\sigma_n|\,\alpha^2\varepsilon^{2/3} + O(\varepsilon)$, where α is a negative constant that explicitly relates to the physical parameters and σ_n is either the n-th extremum or the n-th zero of the Airy function Ai, depending on the kind (respectively, bosons or fermions) of the two heavy particles. Additionally, we prove that the essential spectrum coincides with the half-line $[-\frac{\alpha^2}{4+\varepsilon^2}\,, +\infty)$.

This is a joint work with Andrea Posilicano and Hamidreza Saberbaghi.

Monday March 16, 14:00 – Colette De Coster

Constant sign and sign changing NLS ground states on metric graphs

In this talk, we investigate existence and nonexistence of positive and nodal solutions for the nonlinear Schrödinger equation on metric graphs

$$\begin{cases} -u'' + \lambda u = |u|^{p-2}u & \text{on every edge } e \in \mathbb{E} \\ u \text{ is continuous} & \text{on } \mathcal{G} \\ u(\mathrm{v}) = 0 & \text{at every vertex } \mathrm{v} \in \mathbb{V}_0 \\ \sum_{e \succ \mathrm{v}} u_e'(\mathrm{v}) = 0 & \text{at every vertex } \mathrm{v} \in \mathbb{V} \setminus \mathbb{V}_0 \end{cases}$$

where $p > 2$, $\lambda > 0$, $\mathcal{G}$ is a metric graph.

We first introduce variational methods, based on the so-called *Nehari manifold*, used to obtain particular solutions known as *ground states* or *nodal ground states*.

For noncompact graphs with finitely many edges, we detect purely topological sharp conditions preventing the existence of ground states or of nodal ground states. We also investigate analogous conditions of metrical nature. The negative results are complemented by several sufficient conditions to ensure existence of these particular solutions, either of topological or metrical nature, or a combination of the two.

We then conclude with informations on possible extension of the method.

This is based mainly on joint work with Simone Dovetta (Politecnico di Torino (Italy)), Damien Galant (U. Brown (USA)), Enrico Serra (Politecnico di Torino (Italy)), Christophe Troestler (UMons (Belgium)).

Monday March 16, 15:00 – Anna Rozanova-Pierrat

On the existence of optimal shapes in architecture

We consider shape optimization problems for elasticity systems in architecture. A typical objective in this context is to identify a structure of maximal stability that is close to an initially proposed one. For structures without external forces on varying parts, classical methods allow proving the existence of optimal shapes within well-known classes of bounded uniformly Lipschitz domains. We discuss this for maximally stable roof structures. We then introduce a more general framework that includes external forces on varying parts (for instance, caused by loads of snow on roofs) and prove the existence of optimal shapes, now in a subclass of bounded uniformly Lipschitz domains, endowed with generalized surface measures on their boundaries. These optimal shapes realize the infimum of the corresponding energy of the system. Generalizing further to yet another, very new framework, now involving classes of bounded uniform domains with fractal measures on their boundaries, we finally prove the existence of optimal architectural shapes that actually realize the minimum of the energy. As a by-product we establish the well-posedness of the elasticity system on such domains. In an auxiliary result we show the convergence of energy functionals along a sequence of suitably converging domains. This result is helpful for an efficient approximation of an optimal shape by shapes that can be constructed in practice.

Monday March 16, 15:30 – Simon Becker

Quantum Graphs, Tight-Binding Models, and continuous Schrödinger operators in magnetic fields

We review connections between quantum graphs, tight-binding models, and continuous Schrödinger operators in magnetic fields. In this setting, topology and magnetic flux lead naturally to effective Harper-type operators, whose spectra exhibit fractal structures and metal–insulator transitions. These results highlight how quantum graph and continuum models provide a unified framework for understanding phenomena such as Hofstadter-type spectra and localization–delocalization transitions in condensed matter systems.

Monday March 16, 16:00 – Mats-Erik Pistol

Isospectral quantum graphs

We have studied a variety of isospectral quantum graphs using exact computer algebra, where different permutation invariant selfadjoint boundary conditions have been imposed. There are four families of such boundary conditions, each parametrized by a real parameter. We found one example of an isospectral pair under all four families of permutation-invariant boundary conditions for any value of the relevant parameter. Our software also allows the generation of Titchmarsh-Weyl m-functions which we give for a large set of quantum graphs. Knowledge of the Titchmarsh-Weyl m-function allows the generation of an infinite number of isospectral pairs in many cases. We will show an example of two graphs which are isospectral duals, in a sense that will be made precise. Roughly, one can switch boundary conditions for the two graphs and they remain isospectral. Our software allows the visualisation of eigenfunctions for all permutation invariant selfadjoint boundary conditions.

Tuesday March 17, 09:00 – Julie Rowlett

Crystals, polytopes, and spectrum

The planar crystallographic groups were classified in the late 19th century. A century later, Bérard proved that the fundamental domains of all such groups satisfy a very special analytic property: all Dirichlet eigenfunctions for the Laplacian are trigonometric functions. Bérard also gave a recipe for calculating the corresponding spectrum. In 2008, McCartin proved that in two dimensions, this special analytic property is equivalent to a special geometric property. I'll explain these results and how they can be generalized to all dimensions. The talk is based on joint work with M. Blom, H. Nordell, O. Thim, J. Vahnberg, as well as joint work with G. Mårdby and work-in-progress with Mårdby and F. Francesconi.

Tuesday March 17, 10:00 – Davide Buoso

Spectral convergence analysis for the Reissner-Mindlin system

The Reissner-Mindlin model for the load of a plate can be thought of as a generalization of the classical Kirchhoff-Love model that leads to a bi- harmonic problem. After introducing the Stummel-Vainikko set up for the convergence of operators on varying Banach spaces, we will show that the Reissner-Mindlin operator converges to the biharmonic operator in the norm resolvent convergence as the thickness goes to zero, in particular implying the convergence of eigenvalues and spectral projections. Based on a joint work with Francesco Ferraresso.

Tuesday March 17, 11:00 – Hugo Tavares

Existence and regularity properties of spectral optimal partition problems

In this talk, I will discuss several classes of spectral partition problems inside a box, with and without measure constraints. I will give an overview of general results on the existence of solutions, the optimal regularity of associated eigenfunctions, and free boundary regularity, highlighting the similarities and differences between problems with and without measure constraints.

Tuesday March 17, 11:30 – Jaume De Dios Pont

The hot spots conjecture is false. How false is it?

A homogeneous, insulated object with a non-uniform initial temperature will eventually reach thermal equilibrium. At which point in the object does the temperature take the longest to reach this equilibrium? Intuition tells us that points "in the middle" of the material will generically reach this equilibrium temperature faster, and points "far from the middle" (i.e. at the boundary) should take more time to reach this temperature. Rauch's "hot spots" conjecture is a formalization of this intuition.

We now know the conjecture is false for many reasonable classes of domains. But how false is it? Steinerberger introduced the Hot Spots ratio in order to measure the degree of failure of the conjecture, and proved that it is bounded uniformly over all dimensions. In this talk we will find the exact value of this ratio in every dimension, and understand the shape of domains for which the hot spots conjecture conjecture is "as false as possible".

Tuesday March 17, 13:30 – Alexandre Girouard

Upper bounds for the second nonzero eigenvalue of the Laplacian via folding and conformal volume

We prove an upper bound for the volume-normalized second nonzero eigenvalue of the Laplace operator on closed Riemannian manifold, in terms of the conformal volume. This bound provides effective upper bound for a large class of manifolds, thereby generalizing many known results.

The proof uses the spherical cap folding mechanism originating in work of Nadirashvili in combination with the definition of the conformal volume of Li and Yau. This leads to very convenient admissible trial functions for the min-max characterisation of the second non-zero eigenvalue.

This is based on joint work with Mehdi Eddaoudi.

Tuesday March 17, 14:30 – Gloria Paoli

Serrin's overdetermined problems for Hessian operators: symmetry and stability

This work is in collaboration with Nunzia Gavitone, Alba Lia Masiello, and Giorgio Poggesi. We will provide a new proof of the symmetry for overdetermined Serrin-type problems for Hessian equations (originally established by Brandolini, Nitsch, Salani, and Trombetti), and quantitative stability estimates. In fact, leveraging the analysis performed in the classical case (i.e., with classical mean curvature and classical Laplacian) by Magnanini and Poggesi in a series of papers, we will extend their approach to the higher order setting (i.e., with k-order mean curvature and the k-Hessian operator, for $k \geq 1$) achieving various quantitative estimates of closeness to the symmetric configuration.

Wednesday March 18, 09:00 – Martina Vittorietti

Statistical Inference for Poisson-Laguerre tessellation: inversion, reconstruction and stereology

In this talk, first we consider statistical inference for Poisson-Laguerre tessellations in $\mathbb{R}^d$. The object of interest is a distribution function F which describes the distribution of the arrival times of the generator points. The function F uniquely determines the intensity measure of the underlying Poisson process.

We consider two scenarios: first, we assume to know the points of the Poisson process which generate non-empty cells and the actual cells corresponding to these points and construct two nonparametric estimators. The proposed estimators are proven to be strongly consistent, as the observation window expands unboundedly to the whole space.

Then, we assume to do not know the points and to want to retrieve them inverting the tessellation. We provide sufficient conditions for consistently inverting the observed Poisson-Laguerre tessellation, as the observation window expands unboundedly to the whole space. The performance of the two estimators, previously proposed is investigated also in this setting. Finally, we consider a stereological setting, where one is interested in estimating the distribution function associated with the Poisson process of a higher dimensional Poisson-Laguerre tessellation, given that a corresponding sectional Poisson-Laguerre tessellation is observed.

Wednesday March 18, 10:00 – Daniela Flimmel

Recent advances in hyperuniformity

Hyperuniformity describes many-body systems in which large-scale density fluctuations are anomalously suppressed, placing them between crystals and typical disordered materials. This feature was long implicit in studies of crystals and quasicrystals. In 2003, the concept was formalized in [1], where the authors unified diverse examples, from atomic systems to photonic materials and cosmology, under a common framework of hyperuniformity. Since then, hyperuniformity has become a central idea in condensed matter physics, materials science, and soft matter, with applications to wave control, transport, and biological organization. Mathematical research has expanded rapidly as well, leading to significant results in the theory of point processes, optimal transport, discrete geometry, and random matrix theory. In this talk, we review recent advances and formulate several open problems.

[1] Torquato, S. and Stillinger, F. H. (2003): *Local density fluctuations, hyperuniformity, and order metrics*, Phys. Rev. E 68, 041113.

Wednesday March 18, 11:00 – Eviatar B. Procaccia

Multi-cluster diffusion limited aggregation

The Cluster-cluster model was introduced by Meakin et al in 1984. Each $x \in Z^d$ starts with a cluster of size 1 with probability $p \in (0, 1]$ independently. Each cluster $\mathcal{C}$ performs a continuous-time simple random walk with rate $|\mathcal{C}|^{-\alpha}$. In this talk I will present exact cluster size growth in dimension 1, and condensation conditions in any dimension. For slightly negative α we show that condensation is sensitive to the starting position.

Wednesday March 18, 11:30 – Luca Makowiec

Random Spanning Trees in Random Environment

We introduce Random Spanning Trees in Random Environment (RSTRE), a disordered system on spanning trees that interpolates between the Uniform Spanning Tree (UST) and Minimum Spanning Tree (MST). Our primary goal is to study how the diameter of a (typical) spanning tree depends on the inverse temperature, as this is the first step towards establishing convergence to a non-trivial scaling limit. We first discuss the model on expander graphs and on boxes in the lattice, and then turn to RSTREs on the complete graph. In this setting, we identify different "scales" of the disorder variables that influence the diameter of the tree, giving us a low and high disorder phase. We end the talk by giving a conjecture about the diameter in the intermediate phase.

Wednesday March 18, 13:30 – Viktor Beneš

Two-scale stochastic modeling and simulation in polycrystalline materials under loading

Methods of probability and statistics are commonly used when dealing with some phenomena in the microstructure of metals. We demonstrate an application of stochastic geometry, spatial statistics, stochastic simulation and stereology to the modeling of deformation twinning in polycrystalline materials when a specimen is exposed to mechanical loading. Specifically, the deformation twinning gives rise to a nested tessellation marked by crystallographic orientations, where the subcells are parallel twin lamellae and interlamellar spaces. In the next step a numerical simulation of the stress and strain tensor fields resulting from virtual deformation twinning is provided over the specimen. Thus jointly we deal with a model on two scales. Finally the total strain energy density is evaluated and its sensitivity to selected key parameters is investigated using statistical methods.

Joint work with Luděk Heller (FZÚ CAS), Zbyněk Pawlas and Oleksandr Kornijčuk (CU)

Wednesday March 18, 14:30 – Vesna Gotovac Đogaš

Topological data analysis for random sets

This work paves the way for a methodology for detecting outliers and testing the goodness-of-fit of random sets using topological data analysis. Our approach is based on building a filtration using the sublevel sets of the signed distance function and analyzing various summary functions derived from the persistence diagrams of the resulting persistent homology. Outlier detection is performed using functional depth measures applied to these summary functions. To evaluate goodness-of-fit, we use global envelope tests with summary statistics serving as test statistics. The methodology is supported by a simulation study based on random set germ-grain models and is also applied to real-world data from histological images of mastopathic and breast cancer tissue.

Wednesday March 18, 15:00 – Jussi Behrndt

Spectral theory of two-dimensional Laplacians with oblique Robin boundary conditions

A new type of Robin boundary conditions for the two-dimensional Laplacian is introduced, where besides the normal derivative also the tangential derivative along the boundary is taken into account. This leads to unexpected spectral and regularity properties of the corresponding self-adjoint operators.
This talk is based on joint work with Markus Holzmann and Georg Stenzel.

Thursday March 19, 09:00 – José M. Mazón

Double Nonlinear Diffusion Equations on Metric Graphs

In this lecture we study existence and uniqueness of solutions for a very general class of double nonlinear diffusion equations on metric graphs. Some important particular cases of our results are the Porous Medium Equation and the evolution equation for the p-Laplacian, but we also consider the case in that diffusion change from one edge to another. The problem is moreover studied under non-homogeneus Neumann-Kirchhoff conditions on the vertices of the graph.

Joint work with J. Toledo.

Thursday March 19, 10:00 – Ozlem Defterli

Mathematical Modelling of Complex Dynamics in Biology via Non-Local Operators

This study explores recent developments in the mathematical modeling of complex biological systems, emphasizing the role of fractional (non-local) operators. Through a comparative analysis of classical and fractional models, it demonstrates how memory-dependent dynamics enhance the understanding of cancer growth and infectious disease spread. The findings highlight the potential of fractional modeling to improve and optimize treatment strategies.

Thursday March 19, 11:00 – Liviu Ignat

Convection diffusion problems with non-integrable initial data

In this talk we present some recent result about the long time behavior of the solutions to the one-dimensional diffusion problem with nonlinear convection

$$u_t - u_{xx} + (F(u))_x = 0 \quad \text{in } Q := (0,\infty)\times\mathbb{R}, \qquad u(0) = u_0 \quad \text{on } \mathbb{R},$$

where

$$F(u) := a\,|u|^{q-1}u, \qquad a \in \{\pm 1\}, \qquad q > 1,$$

and $u_0 \in L^\infty(\mathbb{R}^N)$ but not necessarily integrable. The prototype of the initial data we have in mind is

$$u_0(x) \simeq \frac{k_\pm}{|x|^\gamma} \quad \text{as } x \to \pm\infty, \qquad 0 < \gamma \leq 1.$$

In contrast with the case of integrable data where critical exponent governing the long time bahaviour is $q = 2$, here $q_c = 1 + \frac{1}{\gamma}$. We obtain the main term in the asymptotic expansion of the solutions for large time.

This is a joint work with Fernando Quiros (Universidad Autonoma de Madrid).

Thursday March 19, 11:30 – Antonio Esposito

Nonlocal equations on co-evolving graphs

In this talk we consider a class of evolutionary equations on co-evolving, infinite, graphs, motivated by applications in data science and opinion dynamics. We shall discuss an interesting link with a class of nonlinear continuity equations, whose vector fields depend on the graphs considered. More precisely, weak solutions of the so-called graph-continuity equation are shown to be the push-forward of their initial datum through the flow map solving the associated characteristics' equation, which depends on the co-evolving graph considered. This connection can be used to prove contractions in a suitable distance, although the flow on the graphs requires a too limiting assumption on the overall flux. Therefore, we consider upwinding dynamics on graphs with pointwise and monotonic velocity and prove long-time convergence of the solutions towards the uniform mass distribution. The talk is based on joint works with J. A. Carrillo and L. Mikolás (Oxford).

Thursday March 19, 13:30 – Giuseppe Savaré

The De Giorgi Variational Principle for Gradient Flows: A Direct Approach

De Giorgi's variational principle provides a global-in-time characterization of gradient flows of a functional ϕ as curves minimizing an energy–dissipation functional. In the classical Hilbertian setting, this functional takes the form

$$\mathcal{J}(x) := \phi(x(T)) + \int_0^T \left(\tfrac{1}{2}\|\dot{x}(t)\|^2 + \tfrac{1}{2}\|\nabla\phi(x(t))\|^2 \right) dt,$$

for curves x satisfying $x(0) = x_0$.

While the existence of minimizers of $\mathcal{J}$ is relatively easy to establish (under suitable compactness and closure assumptions), the original formulation of De Giorgi's principle does not immediately imply that the minimum value of $\mathcal{J}$ coincides with the initial energy $\phi(x_0)$. This identification is crucial, since only minimizers attaining this value correspond to genuine solutions of the gradient flow.

In this talk I will present a direct variational approach showing how to prove that

$$\min \mathcal{J} = \phi(x_0),$$

without relying on time-discretization or iterative constructions such as the minimizing movements scheme. The key idea is to replace iterative arguments with a global method based on relaxation, convexification, and duality, allowing one to identify the value of the minimum directly at the continuous level.

Although the presentation focuses on the simplest case of gradient flows in Hilbert spaces, the same strategy extends to more general frameworks, including doubly nonlinear equations in reflexive Banach spaces, metric gradient flows, and time-dependent energies.

Joint work with Alessandro Pinzi and Filippo Riva.

Thursday March 19, 14:30 – Sara Farinelli

On the stability of the JKO operator under different notions of convexity

Given a metric space (X,d) and a functional $\mathcal{F}$ defined on it, a relevant role in the theory of gradient flows associated with $\mathcal{F}$ is played by the *proximity operator*

$$P_{\mathcal{F}}(x) := \arg\min_{y} \left\{ \frac{1}{2} d^2(x,y) + \mathcal{F}(y) \right\}.$$

In the setting of the Wasserstein space, that is $(X,d) = (\mathcal{P}_2(\mathbb{R}^d), W_2)$, the proximity operator is often called *JKO operator*. Motivated from the fact that if (X,d) is a Hilbert space then $x \mapsto P_{\mathcal{F}}(x)$ is non-expansive when $\mathcal{F}$ is convex, the non-expansivity of the *JKO* operator has been a topic of investigation. It has been proved recently by Cavagnari-Savaré-Sodini that if $\mathcal{F}$ is *totally convex* then non-expansivity holds. Various partial results are present in literature assuming weaker notions of convexity. In this talk we review these contributions and we present some generalizations under the assumption that $\mathcal{F}$ is convex on generalized geodesics. This is based on a joint work in preparation with Di Marino and Naldi.