Spectral convergence analysis for the Reissner-Mindlin system

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A “toy” problem

Let us give a look at the Robin problem for the Laplacian

$$\begin{cases} -\Delta u = f, & \text{in } \Omega, \\ \partial_\nu u + \alpha u = 0, & \text{on } \partial\Omega, \end{cases}$$

with weak formulation

$$\int_\Omega \nabla u \nabla v + \int_{\partial\Omega} \alpha u v = \int_\Omega f v,$$

with energy space $H^1(\Omega)$.

$\mathcal{E}$-convergence

Definition

Let $\{\mathcal{H}_\delta\}_{\delta\in[0,\bar{\delta})}$ be a family of Hilbert spaces. We assume that there exists a family of linear operators $\mathcal{E}_\delta \in \mathcal{L}(\mathcal{H}_0, \mathcal{H}_\delta)$ called connecting system such that, for all $u_0 \in \mathcal{H}_0$,

$$\|\mathcal{E}_\delta u_0\|_{\mathcal{H}_\delta} \to \|u_0\|_{\mathcal{H}_0}, \quad \text{as } \delta \to 0^+.$$

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$$\|\mathcal{E}_\delta u_0\|_{\mathcal{H}_\delta} \to \|u_0\|_{\mathcal{H}_0}\,, \quad \text{as } \delta \to 0^+.$$

- *Let $u_\delta \in \mathcal{H}_\delta$. We say that u_δ $\mathcal{E}$-converges to u_0 if $\|u_\delta - \mathcal{E}_\delta u_0\|_{\mathcal{H}_\delta} \to 0$ as $\delta \to 0^+$. We write $u_\delta \xrightarrow{\mathcal{E}} u_0$.*

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$$\|\mathcal{E}_\delta u_0\|_{\mathcal{H}_\delta} \to \|u_0\|_{\mathcal{H}_0}, \quad \text{as } \delta \to 0^+.$$

• Let $u_\delta \in \mathcal{H}_\delta$. We say that u_δ $\mathcal{E}$-converges to u_0 if $\|u_\delta - \mathcal{E}_\delta u_0\|_{\mathcal{H}_\delta} \to 0$ as $\delta \to 0^+$. We write $u_\delta \xrightarrow{\mathcal{E}} u_0$.
• Let $B_\delta \in \mathcal{L}(\mathcal{H}_\delta)$. We say that B_δ $\mathcal{E}\mathcal{E}$-converges to B_0 as $\delta \to 0^+$ if $B_\delta u_\delta \xrightarrow{\mathcal{E}} B_0 u_0$ whenever $u_\delta \xrightarrow{\mathcal{E}} u_0$. We write $B_\delta \xrightarrow{\mathcal{E}\mathcal{E}} B_0$.

$\mathcal{E}$-convergence

Definition (continued)

- *Let $B_\delta \in \mathcal{L}(\mathcal{H}_\delta)$. We say that B_δ compactly converges to B_0 as $\delta \to 0^+$, and we write $B_\delta \xrightarrow{C} B_0$, if the following two conditions are satisfied:*
  (a) *$B_\delta \xrightarrow{\mathcal{E}\mathcal{E}} B_0$ as $\delta \to 0^+$;*
  (b) *for any family $u_\delta \in \mathcal{H}_\delta$ such that $\|u_\delta\|_{\mathcal{H}_\delta} = 1$ for all $\delta \in (0, \bar{\delta})$, there exists a subsequence $\{B_{\delta_k} u_{\delta_k}\}_{k\in\mathbb{N}}$ with $\delta_k \to 0^+$ as $k \to +\infty$, and $u_0 \in \mathcal{H}_0$ such that $B_{\delta_k} u_{\delta_k} \xrightarrow{\mathcal{E}} u_0$ as $k \to +\infty$.*

Spectral convergence

Theorem

Let A_δ, $\delta \in [0, \bar{\delta})$ *be a family of positive, self-adjoint differential operators on* $\mathcal{H}_\delta$ *with domain* $\mathcal{D}(A_\delta) \subset \mathcal{H}_\delta$.

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Assume moreover that

- *the resolvent operator* $B_\delta := A_\delta^{-1}$ *is compact for all* $\delta \in [0, \bar{\delta})$;
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Then, if λ_0 *is an eigenvalue of* A_0, *there exists a sequence of eigenvalues* λ_δ *of* A_δ *such that* $\lambda_\delta \to \lambda_0$ *as* $\delta \to 0^+$.

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Then, if λ_0 *is an eigenvalue of* A_0*, there exists a sequence of eigenvalues* λ_δ *of* A_δ *such that* $\lambda_\delta \to \lambda_0$ *as* $\delta \to 0^+$*.*
Conversely, if λ_δ *is an eigenvalue of* A_δ *for all* $\delta \in (0, \bar{\delta})$*, and* $\lambda_\delta \to \lambda_0$*, then* λ_0 *is an eigenvalue of* A_0*.*

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Then, if λ_0 *is an eigenvalue of* A_0*, there exists a sequence of eigenvalues* λ_δ *of* A_δ *such that* $\lambda_\delta \to \lambda_0$ *as* $\delta \to 0^+$*. Conversely, if* λ_δ *is an eigenvalue of* A_δ *for all* $\delta \in (0, \bar{\delta})$*, and* $\lambda_\delta \to \lambda_0$*, then* λ_0 *is an eigenvalue of* A_0*. The generalised eigenspaces (resp. the spectral projections) of* A_δ *at* λ_0 *compactly converge to the* λ_0*-eigenspace (resp. the* λ_0*-spectral projection) of* A_0 *as* $\delta \to 0^+$*.*

Generalised norm resolvent convergence

Definition

Let $(\mathcal{H}_\delta)_\delta, \mathcal{H}_0$ *be Hilbert spaces and let* $((\mathcal{H}_\delta)_\delta, \mathcal{H}_0, (\mathcal{E}_\delta)_\delta)$ *be a connecting system in the sense of Vainikko. Let* $(A_\delta)_\delta$ *be a family of closed linear operators,* A_δ *acting in* $\mathcal{H}_\delta$ *for each* $\delta > 0$, A_0 *acting in* $\mathcal{H}_0$. *Let* $\lambda \in \left(\bigcap_\delta \rho(A_\delta) \cap \rho(A_0)\right)$. *We say that* A_δ *converges to* A_0 *with respect to (Vainikko) generalised norm resolvent convergence if*

$$\sup_{f_0 \in \mathcal{H}_0} \frac{||(A_\delta - \lambda)^{-1}\mathcal{E}_\delta f_0 - \mathcal{E}_\delta (A_0 - \lambda)^{-1} f_0||_{\mathcal{H}_\delta}}{||f_0||_{\mathcal{H}_0}} \to 0$$

as $\delta \to 0^+$.

Generalised norm resolvent convergence

The notion of generalised norm resolvent convergence is important because it implies the existence of a modulus of continuity ω such that

$$||(A_\delta - \lambda)^{-1}\mathcal{E}_\delta - \mathcal{E}_\delta(A_0 - \lambda)^{-1}|| \leq \omega(\delta),$$

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which in turn implies rates of convergence e.g., for eigenvalues.

Eigenvalue convergence

Theorem

Let $\lambda_0 \in \sigma(A_0)$ *be an eigenvalue of multiplicity* $m \geq 1$ *and let* $\lambda_\delta^i \in \sigma(A_\delta)$, $i = 1, \dots, m$, *be such that* $\lambda_\delta^i \to \lambda_0$ *as* $\delta \to 0^+$. *Then there exists a constant* C *such that*

$$\sum_{i=1}^{m} |\lambda_\delta^i - \lambda_0| \leq C\omega(\delta),$$

and C *can be chosen to depend only on* λ_0, A_0, *and the connecting system* $(\mathcal{E}_\delta)_\delta$.

The Kirchhoff-Love model

Given a plate $\Omega \times (-\epsilon/2, \epsilon/2)$ of cross-section $\Omega \subseteq \mathbb{R}^2$ and negligible thickness ϵ, if we apply a load f the displacement u will be given by

$$\Delta^2 u = f, \quad \text{in } \Omega.$$

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This equation will be complemented with suitable boundary conditions.

The Reissner-Mindlin model

Given a plate $\Omega \times (-t/2, t/2)$ of thickness t, if we apply a load f and a couple F the displacement w and the fiber rotation β will be given by

$$\begin{cases} -\frac{\mu_1}{12}\Delta\beta - \frac{\mu_1+\mu_2}{12}\nabla(\operatorname{div}\beta) - \frac{\mu_1 k}{t^2}(\nabla w - \beta) = \frac{t^2}{12}F, & \text{in } \Omega, \\ -\frac{\mu_1 k}{t^2}(\Delta w - \operatorname{div}\beta) = f, & \text{in } \Omega. \end{cases}$$

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This system will be complemented with suitable boundary conditions.

Two different models

These two models are very different and used in different situations, yet they are intimately linked. It is somewhat known that, as $t \to 0^+$, the Reissner-Mindlin problem converges to the bilaplacian and that the fiber rotation should be close to the gradient of the displacement, i.e., $\beta = \nabla w$.

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Let us now present in detail the RM system.

RM: weak formulation

Let $\Omega \subseteq \mathbb{R}^N$, $f \in L^2(\Omega)$, $F \in L^2(\Omega)^N$. The weak formulation of the RM system reads

$$\frac{E}{12(1-\sigma^2)} \int_\Omega \Big((1-\sigma)\varepsilon(\beta) : \varepsilon(\eta) + \sigma \operatorname{div}(\beta) \operatorname{div}(\eta) \Big)\, dx \\ + \frac{Ek}{2(1+\sigma)t^2} \int_\Omega (\nabla w - \beta) \cdot (\nabla v - \eta) dx \\ = \int_\Omega \left(fv + \frac{t^2}{12} F \cdot \eta \right) dx,$$

for all $(\eta, v) \in V \times W$, where $H^1_0(\Omega)^N \subseteq V \subseteq H^1(\Omega)^N$ and $H^1_0(\Omega) \subseteq W \subseteq H^1(\Omega)$.

Boundary conditions

Hard clamped boundary conditions. In this case $V = (H_0^1(\Omega))^N$, $W = H_0^1(\Omega)$, producing the following

$$\begin{cases} \beta = 0 = w, \quad \text{on } \partial\Omega. \end{cases}$$

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Soft clamped boundary conditions. In this case $V = \{\Phi \in (H^1(\Omega))^N : \Phi \cdot \nu = 0 \text{ on } \partial\Omega\}$, $W = H_0^1(\Omega)$, producing the following

$$\begin{cases}(\varepsilon(\beta)\nu)_{\partial\Omega} = 0, & \text{on } \partial\Omega,\\ w = 0 = \beta \cdot \nu, & \text{on } \partial\Omega.\end{cases}$$

Boundary conditions

Hard simply supported boundary conditions. In this case $V = \{\Phi \in (H^1(\Omega))^N : (\Phi)_{\partial\Omega} = 0 \text{ on } \partial\Omega\}$, $W = H_0^1(\Omega)$, producing the following

$$\begin{cases} (1-\sigma)\frac{\partial\beta}{\partial\nu} \cdot \nu + \sigma \operatorname{div} \beta = 0, & \text{on } \partial\Omega, \\ w = 0 = (\beta)_{\partial\Omega}, & \text{on } \partial\Omega. \end{cases}$$

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Soft simply supported boundary conditions. In this case $V = (H^1(\Omega))^N$, $W = H^1_0(\Omega)$, producing the following

$$\begin{cases} (1-\sigma)\frac{\partial\beta}{\partial\nu}\cdot\nu + \sigma \operatorname{div}\beta = 0, & \text{on } \partial\Omega, \\ (\varepsilon(\beta)\nu)_{\partial\Omega} = 0, & \text{on } \partial\Omega, \\ w = 0, & \text{on } \partial\Omega. \end{cases}$$

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(Free) Neumann boundary conditions. In this case $V = H^1(\Omega)^N$, $W = H^1(\Omega)$, producing the following

$$\begin{cases} (1-\sigma)\frac{\partial \beta}{\partial \nu} \cdot \nu + \sigma \operatorname{div} \beta = 0, & \text{on } \partial\Omega, \\ (\varepsilon(\beta)\nu)_{\partial\Omega} = 0, & \text{on } \partial\Omega, \\ \frac{\partial w}{\partial \nu} - \beta \cdot \nu = 0, & \text{on } \partial\Omega. \end{cases}$$

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Weak Neumann boundary conditions. In this case $V = \{\Phi \in (H^1(\Omega))^N : (\Phi)_{\partial\Omega} = 0 \text{ on } \partial\Omega\}$, $W = H^1(\Omega)$, producing the following

$$\begin{cases} (1-\sigma)\frac{\partial \beta}{\partial \nu} \cdot \nu + \sigma \operatorname{div} \beta = 0, & \text{on } \partial\Omega, \\ (\beta)_{\partial\Omega} = 0, & \text{on } \partial\Omega, \\ \frac{\partial w}{\partial \nu} - \beta \cdot \nu = 0, & \text{on } \partial\Omega. \end{cases}$$

Boundary conditions

Hard rigid boundary conditions. In this case $V = H_0^1(\Omega)^N$, $W = H^1(\Omega)$, producing the following

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Boundary conditions

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Soft rigid boundary conditions. In this case $V = \{\Phi \in (H^1(\Omega))^N : \Phi \cdot \nu = 0 \text{ on } \partial\Omega\}$, $W = H^1(\Omega)$, producing the following

$$\begin{cases} (\varepsilon(\beta)\nu)_{\partial\Omega} = 0, & \text{on } \partial\Omega, \\ \frac{\partial w}{\partial \nu} = 0 = \beta \cdot \nu, & \text{on } \partial\Omega. \end{cases}$$

Thin plates

Now let us move to the limit as $t \to 0^+$. In the literature only the case $N = 2$ has been considered so far, with a proof of convergence of the eigenvalues only in the hard clamped case.

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Now let us move to the limit as $t \to 0^+$. In the literature only the case $N = 2$ has been considered so far, with a proof of convergence of the eigenvalues only in the hard clamped case. In order to avoid issues with the kernel, we add the identity to the RM problem and show its convergence to the bilaplacian as

$$\frac{E}{12(1-\sigma^2)}\Delta^2 u + u = f.$$

Thin plates

In particular, the weak formulation of this problem reads

$$\frac{E}{12(1-\sigma^2)}\int_\Omega \left((1-\sigma)D^2u : D^2v + \sigma\Delta u\Delta v\right)dx \\ + \int_\Omega uvdx = \int_\Omega fvdx.$$

Thin plates

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Let us see what are the corresponding boundary conditions depending on the energy space.

Bilaplacian boundary conditions

- Hard and soft clamped conditions will converge to clamped (Dirichlet) boundary conditions:

$$u = \frac{\partial u}{\partial \nu} = 0.$$

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- Hard and soft clamped conditions will converge to clamped (Dirichlet) boundary conditions:

$$u = \frac{\partial u}{\partial \nu} = 0.$$

- Hard and soft simply supported conditions will converge to hinged (Navier) boundary conditions:

$$u = (1 - \sigma)\frac{\partial^2 u}{\partial \nu^2} + \sigma \Delta u = 0.$$

Bilaplacian boundary conditions

- Soft rigid conditions will converge to intermediate (Kuttler-Sigillito) boundary conditions:

$$\frac{\partial u}{\partial \nu} = \frac{\partial \Delta u}{\partial \nu} + (1 - \sigma)\, \mathrm{div}_{\partial\Omega}(\nu \cdot D^2 u)_{\partial\Omega} = 0.$$

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- Soft rigid conditions will converge to intermediate (Kuttler-Sigillito) boundary conditions:

$$\frac{\partial u}{\partial \nu} = \frac{\partial \Delta u}{\partial \nu} + (1-\sigma)\,\mathrm{div}_{\partial\Omega}(\nu \cdot D^2 u)_{\partial\Omega} = 0.$$

- Free Neumann conditions will converge to free (Neumann) boundary conditions:

$$(1-\sigma)\frac{\partial^2 u}{\partial \nu^2} + \sigma \Delta u = \frac{\partial \Delta u}{\partial \nu} + (1-\sigma)\,\mathrm{div}_{\partial\Omega}(\nu \cdot D^2 u)_{\partial\Omega} = 0.$$

Bilaplacian boundary conditions

The remaining two sets of boundary conditions will lead to non-standard boundary value problems. If Ω is smooth enough (e.g., $\partial\Omega \in C^2$), hard rigid boundary conditions will converge to the following

$$\begin{cases} \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega, \\ u \text{ constant on every connected component of } \partial\Omega, \end{cases}$$

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$$\begin{cases} \frac{\partial u}{\partial \nu} = 0 \qquad \text{on } \partial\Omega, \\ u \text{ constant on every connected component of } \partial\Omega, \end{cases}$$

while weak Neumann boundary conditions will converge to the following

$$\begin{cases} (1-\sigma)\frac{\partial^2 u}{\partial \nu^2} + \sigma \Delta u = 0 \quad \text{on } \partial\Omega, \\ u \text{ constant on every connected component of } \partial\Omega. \end{cases}$$

RM converges to KM

Let A_t be the operator associated with the RM system with thickness t, and A_0 that associated with the bilaplacian.

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Let $B_t = A_t^{-1}$ *and* $B_0 = A_0^{-1}$. *Then* $B_t \xrightarrow{C} B_0$ *as* $t \to 0^+$. *In particular the eigenvalues* $\lambda_k(t) \in \sigma(A_t)$ *converge to* $\lambda_k(0) \in \sigma(A_0)$ *as* $t \to 0^+$.

RM converges to KM

The proof consists of three steps.
Step 1. The solution (β_t, w_t) is H^1-bounded, in particular it has a strong L^2-limit as $t \to 0^+$. This is done through a fine study of norm estimates.

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Step 3. The convergence is actually compact.

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Step 2. The limit w_0 satisfies the bilaplacian problem.
Step 3. The convergence is actually compact.
This concludes the proof.

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A_t converges to A_0 in generalised norm resolvent convergence as $t \to 0^+$.

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Even though the proof is not difficult, it is non constructive and therefore yields no information whatsoever on the rate of convergence. Other techniques in two dimensions provide some information in the hard clamped case, but the scenario is not at all clear for the general case.

Thank you for your attention