Report on **CA24122 (mSPACE)** sessions

20 – 21 April 2026, CIRM Luminy, FR

Two afternoons (20 and 21 April) during the conference

Mathematics of the physics with non-self-adjoint operators

organised in the *Centre International de Rencontres Mathématiques* in Luminy (FR) on 20 – 24 April 2026 (with 89 participants) were devoted to the mSPACE Action. All four organisers

- Lucrezia Cossetti (University of the Basque Country, ES)
- Borbala Gerhat (Institute of Science and Technology Austria, AT)
- David Krejčiřík (Czech Technical University in Prague, CZ)
- Petr Siegl (Graz University of Technology, AT)

are members of the Action (WG1, WG2). Below we describe the two afternoon sessions, with emphasis on their connection to mSPACE and its members. In particular, we highlight the roles of the members who received the daily allowance **(DA)**.

Monday 20 April

The first COST session on the afternoon of 20 April 2026 was initiated by a talk given by the Action Chair **Delio Mugnolo (DA)**, who presented the Action and its funding opportunities for members. This introduction was followed by three contributed talks of mSPACE members (chaired by **Delio Mugnolo**). After a coffee break, the afternoon was concluded by a poster session, which was catered using the granted **Local Organiser Support**. Among other participants, five mSPACE members presented a poster. A detailed schedule of the session, as well as the abstracts of the scientific contributions, are listed below.

14.30 – 14.40 : **Delio Mugnolo** (WG1 – WG4)
COST Action CA24122 mSpace

14.45 – 15.10 : Maryna Kachanovska (WG4)
Wave Propagation in a Waveguide Filled with a Cold Strongly Magnetized Plasma (Chair **D. Mugnolo**)

15.15 – 15.40 : Laura Baldelli (WG1, WG2)
Schrödinger operators in electromagnetic fields
(Chair **D. Mugnolo**)

15.45 – 16.10 : Cristian Cazacu (WG1, WG2)
Hardy-Rellich inequalities with distance function to the boundary and some applications (Chair **D. Mugnolo**)

16.15 – 16.30 : Coffee break

16.30 – 18.30 : Poster session (21 posters) including mSPACE members

Catherine Drysdale (WG3) **(DA)**
Computation and Verification of Spectra for Non-Hermitian Systems

Patrick Ebnicher (WG1, WG2)
Neumann Realisation of Schrödinger Operators with Unbounded Complex Potentials

Jan Havel (WG1, WG2)
Local density of states for 2D-Schrödinger operators with δ-point interactions

Mikuláš Kučera (WG1, WG2)
Spectrum of the wave equation with Dirac damping on a compact star graph

Peter Schlosser (WG1, WG2) **(DA)**
The Neumann trace for unbounded Lipschitz domains

Abstracts (talks)

Laura Baldelli : *Schrödinger operators in electromagnetic fields*

In quantum mechanics, Schrödinger operators with electric and magnetic fields model the energy of a nonrelativistic particle subject to electromagnetic forces. In particular, magnetic fields add significant complexity, as their contribution via the vector potential results as imaginary perturbations of the Laplacian. This requires working in complex-valued functional spaces and often limits the applicability of standard analytical methods.

In recent joint work with Roberta Filippucci (University of Perugia, Italy) and David Krejčiřík (Czech Technical University in Prague, Czech Republic), we analyze a critical quasilinear Schrödinger equation involving the magnetic p-Laplacian $\Delta_{A,p}$ and a critical nonlinearity under suitable assumptions.

Cristian Cazacu : *Hardy-Rellich inequalities with distance function to the boundary and some applications*

We present generalizations to the L^p-setting of the Hardy-Rellich inequalities on domains of $\mathbb{R}^N$ with singularities given by the distance function to the boundary. We obtain either sharp constants in bounded domains (where we provide concrete minimizing sequences) or find new bounds for the sharp constant, while also depending on the geometric properties of the domain and its boundary. We also give applications to the

existence and non-existence of solutions for a p-Biharmonic singular problem.

This talk is based on a paper accepted for publication to SIAM. J. Math. Anal., written in collaboration with Teodor Rugina (PhD Student, University of Bucharest. Email: teorugina@yahoo.com).

Maryna Kachanovska : *Wave Propagation in a Waveguide Filled with a Cold Strongly Magnetized Plasma*

We consider the 2D Maxwell equations describing wave propagation in a cold strongly magnetized plasma, posed in a locally perturbed waveguide. Our goal is to analyze the spectrum of the underlying self-adjoint operator, which reduces to studying the behaviour of a boundary-value problem for a PDE with frequency-dependent coefficients. The main difficulty is that, for a range of real frequencies, the principal symbol of the partial differential operator becomes hyperbolic. A similar problem occurs in fluid mechanics ('internal gravity waves'). We show that the limiting absorption principle holds for a range I of frequencies, which depends on the geometric perturbation, by fairly elementary means, namely using the well-posedness of the Cauchy problem for the wave-type equations in moving domains and the positivity of the Dirichlet-to-Neumann maps. Next, we discuss sufficient conditions on the geometric perturbations that ensure the absence of eigenvalues outside of I. This is joint work with Dylan Machado.

Abstracts (posters)

Catherine Drysdale : *Computation and Verification of Spectra for Non-Hermitian Systems*

We establish a deep connection between quantum mechanics and computation, revealing fundamental limitations for algorithms computing spectra, especially in non-Hermitian settings. Introducing the concept of locally trivial pseudospectra (LTP), we show such assumptions are necessary for spectral computation. LTP adapts dynamically to system energies, enabling spectral analysis across a broad class of challenging non-Hermitian problems. Exploiting this framework, we overcome a longstanding obstacle by computing the eigenvalues and eigenfunctions of the imaginary cubic oscillator $H_B = p^2 + ix^3$ with rigorous error bounds and no spurious modes—yielding, to our knowledge, the first such error-controlled result. We confirm, for instance, the 100th eigenvalue as $627.69471224843651135267370290115 36\ldots$. Here, truncation-induced PT-symmetry breaking causes spurious eigenvalues—a pitfall our method avoids, highlighting the link between truncation and physics. Finally, we illustrate the approach's generality via spectral computations for a range of physically relevant operators. This letter provides a rigorous framework linking computational theory to quantum mechanics and offers a precise tool for spectral calculations with error bounds.

Joint work with Matthew Colbrook and Michael T. M. Woodley.

Patrick Ebnicher : *Neumann Realisation of Schrödinger Operators with Unbounded Complex Potentials*

We study Schrödinger operators with complex-valued, possibly unbounded potentials V on unbounded domains subject to Neumann boundary conditions. Assuming that V

has a non-negative real part and suitable L^p_{loc} regularity up to the boundary, we prove that the associated operators are m-accretive.

To rigorously impose the boundary condition at the operator level, we construct a surjective Neumann trace adapted to unbounded domains and unbounded complex potentials.

The proof relies on a complex-valued version of Kato's inequality valid up to the boundary. Our results extend known properties of non-selfadjoint Schrödinger operators beyond classical sectorial and standard form-based approaches.

Jan Havel : *Local density of states for 2D-Schrödinger operators with δ-point interactions*

In physics, the local density of states is a quantity that measures the number of available quantum states per unit energy at a given spatial location. In the case of two-dimensional Schrödinger operators with delta-point interactions, we provide a rigorous definition for positive scattering energies. We also derive formulas showing that the local density of states can be computed either from the Green's function via the limiting absorption principle or from generalised eigenfunctions using the eigenfunction expansion theorem.

Mikuláš Kučera : *Spectrum of the wave equation with Dirac damping on a compact star graph*

We consider the wave equation with a distributional Dirac damping and Dirichlet boundary conditions on a compact interval. It is shown that the spectrum of the corresponding wave operator is fully determined by zeroes of an entire function. Consequently, a considerable change of spectral properties is shown for certain critical values of the damping parameter. We also derive a definitive criterion for the Riesz basis property of the root vectors for an arbitrary placement of a complex-valued Dirac damping. Finally, we consider a generalisation of the problem for compact star graphs and provide insight into the essence of the critical damping constant.

Peter Schlosser : *The Neumann trace for unbounded Lipschitz domains*

For a bounded smooth domains Ω, the Neumann trace operator is classically defined as the bounded extension $\tau_N : H^2(\Omega) \to H^{\frac{1}{2}}(\partial\Omega)$ of the normal derivative acting on smooth functions.

It is then a well known result that this Neumann trace can be extended to an operator $\tau_N : H^1_\Delta(\Omega) \to H^{-\frac{1}{2}}(\partial\Omega)$, via the second Green's identity

$$(\tau_N u, \tau_D v)_{H^{-\frac{1}{2}}(\partial\Omega)\times H^{\frac{1}{2}}(\partial\Omega)} = (\nabla u, \nabla v)_{L^2(\Omega)} - (-\Delta u, v)_{L^2(\Omega)}.$$

Here $H^1_\Delta(\Omega)$ is the space of all function $f \in L^2(\Omega)$, such that also the distributional derivative $\Delta f \in L^2(\Omega)$ is square integrable.

The aim of the present work is to further extend this notion of Neumann trace to:

- Unbounded Lipschitz surfaces;
- Functions $f \in H^{\frac{1}{2}+\varepsilon}(\Omega)$ with Δf only in some dense subset of $L^2(\Omega)$;
- Functions $f \in H^{\frac{1}{2}+\varepsilon}(\Omega)$, with $-\Delta + V$ in some dense subset of $L^2(\Omega)$. Here, the potential V is unbounded, contained in some space $L^p(\Omega)$.

Tuesday 21 April

The second COST session on the afternoon of 21 April 2026 started with an invited talk, followed by a contributed talk of an mSPACE member, both chaired by mSPACE member **Liviu Ignat** (WG1, WG3) **(DA)**. Despite the invited speaker not being a member of the Action, his talk on pseudospectra in general relativity was strongly aligned with the objectives of WG2. After the coffee break, the afternoon was concluded by an open problem session, where **Delio Mugnolo** presented various questions related to non-selfadjoint spectral problems on graphs and hypergraphs (some of which were solved later during the conference). A detailed schedule of the session, as well as the abstracts of the scientific contributions, are listed below.

14.30 – 15.10 : José Luis Jaramillo (related to WG2)
Non-selfadjoint operators in gravity: two case studies
(Chair **L. Ignat**)

15.15 – 15.40 : Nicolas Weber (WG1, WG2)
Eigenvalues of operator families: A commutativity result with an application to 1D Dirac operators (Chair **L. Ignat**)

15.45 – 16.15 : Coffee break

16.15 – 18.00 : Open problem session (**Delio Mugnolo**)

Abstracts

Jose Luis Jaramillo : *Non-selfadjoint operators in gravity: two case studies*

In this talk we present two physical problems, framed in a gravitational setting, defined in terms of non-selfadjoint operators. Specifically, such problems address spectral and dynamical properties of black holes described in general relativity. The first one focuses on a notion of stability for the apparent horizon of a black hole, controlled by the spectrum of the so-called MOTS-stability operator, a non-selfadjoint operator in the rotating case. The characterisation of the qualitative properties of its spectrum is of relevance in different black hole settings. The second problem concerns the linear dynamical regime in the evolution of black holes close to equilibrium. Adopting a hyperboloidal spacetime slicing description to enforce outgoing boundary conditions, the infinitesimal time generator is cast as a non-selfadjoint operator. This feature results in the spectral instability of the so-called quasi-normal mode frequencies, a potential issue for the "black hole spectroscopy programme" in gravitational wave physics. Beyond these spectral aspects, non-modal dynamical effects as growth transients or pseudo-resonances are also the subject of ongoing research in the merger of black holes. These examples provide two open problems in gravitational physics where the input of expertise in non-selfadoint operator theory is crucial. Finally, as a sort of disclaimer, our emphasis in the physics somehow reverses the spirit in the conference title, in what could be rather paraphrased as "physical aspects of the mathematics with non-self-adjoint operators".

Nicolas Weber : *Eigenvalues of operator families: A commutativity result with an application to 1D Dirac operators*

Given two analytic families of bounded operators $A : \Omega \to \mathcal{B}(\mathcal{H}, \mathcal{G})$ and $B : \Omega \to \mathcal{B}(\mathcal{G}, \mathcal{H})$ ($\Omega \subset \mathbb{C}$ is open and $\mathcal{H}$, $\mathcal{G}$ are Hilbert spaces) we prove that the families

$$I_{\mathcal{G}} - AB : \Omega \to \mathcal{B}(\mathcal{G}), \quad I_{\mathcal{H}} - BA : \Omega \to \mathcal{B}(\mathcal{H})$$

have the same eigenvalues of finite type and that their algebraic multiplicities at each respective eigenvalue coincide. The result is applied to the Birman-Schwinger family of a one-dimensional Dirac operator

$$H_\varepsilon = -i\partial_x \sigma_1 + m\sigma_3 + \varepsilon V, \quad \operatorname{dom} H_\varepsilon = H^1(\mathbb{R}, \mathbb{C}^2)$$

in $L^2(\mathbb{R}; \mathbb{C}^2)$ in a weak-coupling setting. We assume that the matrix potential $V : \mathbb{R} \to \mathbb{C}^{2\times 2}$ is relatively-compact and possibly non-Hermitian. In [1] conditions on V were derived under which H_ε has eigenvalues as $\varepsilon \to 0+$ and our result is used to show that these eigenvalues are simple.

This talk is based on joint work with J. Behrndt and P. Siegl.

References:

[1] J.-C. Cuenin, P. Siegl, *Eigenvalues of one-dimensional non-self-adjoint Dirac operators and applications*, Lett. Math. Phys., 108 (2018), 1757-1778.